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Linear Background & Attenuation Corrections for SHARP

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1.

Let the measured signal be given by

$$M(i,t) = G(i)[A(t)S(i) + B(t)] \tag{1}$$

where i denotes a constant point on the sky or an array pixel, and t denotes time.

$$\begin{aligned} B(t) &= \text{background: changes with time, constant across array} && (2) \\ A(t) &= \text{atmospheric attenuation: changes with time, constant across array} && (3) \\ S(i) &= \text{source intensity: varies across the array, constant in time} && (4) \\ G(i) &= \text{detector gain: varies across the array, constant in time} && (5) \end{aligned}$$

Assuming we have already accurately corrected the data for $G(i)$ (the rgm) we drop that factor to yield

$$M(i,t) = S(i)A(t) + B(t). \tag{6}$$

Time averaging yields the quantity:

$$\langle M(i,t)\rangle = S(i)\,\langle A(t)\rangle + \langle B(t)\rangle \tag{7}$$

where $\langle\rangle$ denotes a time average. One expects $\langle B(t)\rangle = 0$ for long times. Substituting back into equation (6) yields

$$\begin{aligned} M(i,t) &= \frac{\langle M(i,t)\rangle}{\langle A(t)\rangle}\,A(t) + B(t) && (8) \\ M(i,t) &= \langle M(i,t)\rangle\,\frac{A(t)}{\langle A(t)\rangle} + B(t) && (9) \end{aligned}$$

This is a linear equation with offset $B(t)$ and slope $A(t)/\langle A(t)\rangle$. If we have a sufficient range of measured source intensities across the array (one measurement per pixel) then we can fit a line to simultaneously determine the atmopsheric attenuation factor and the background for each polarization scan. Typically one would constrain the time average of the background to zero and the time average of the slope to unity.

We then have an estimate of the source intensity:

$$M'(i,t) = S'(i)\,\langle A(t)\rangle = \frac{M(i,t) - B(t)}{A(t)/\langle A(t)\rangle} \tag{10}$$

The solution can be improved through several iterations, replacing $M(i,t)$ in equation (6) with $M'(i,t)$ from equation (10) in each iteration.