Gradient Degeneracy
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Simplified data model:

$$S = G(i)*B(t) + G(i)*M(i, t) + O(i)$$

S = observed signal ("rawdata" in sharcsolve)
G(i) = gain of pixel i ("gain" in sharcsolve)
B(t) = atmospheric background emission at time t ("background" in sharcsolve)
M(i, t) = celestial source map ("image" in sharcsolve)
O(i) = offset of pixel i; electronic in nature ("offset" in sharcsolve)

To demonstrate gradient degeneracy in one axis (x), substitute:

$M' = M - k*x$ (k = constant)
$O' = O + G*k*s*i$ (s = pixel separation)
$B' = B + k*x_0(t)$ (x_0 = position of center of array)

Then:

$$\begin{aligned} S' &= GB' + GM' + O' \\ &= GB + GM + O + Gkx_0 - Gkx + Gksi \\ &= S + Gk(x_0 - x + si) \end{aligned}$$

Since $x(i, t) = x_0(t) + si$:

$$S' = S$$

Therefore, a gradient can be added to source map M without changing the observed signal S.

Observation:
Spatial gradients are removed in the simplified initial solution, which assumes M = 0. More or less, the gradient remains unchanged in later iterations without additional rules (e.g., solution #2 below). In the case of a chopped dual beam image with a bright source, the observed data has a clear gradient in the direction of the chop.

Solutions to gradient degeneracy:

1) Put boundary conditions on map (possible with –rz option in sharcsolve).
2) Remove gradients from background term, i.e., enforce $\langle Bx_0 \rangle = 0$. Now standard in sharcsolve.

Other known degeneracies:

1) arbitrary DC (could appear in M, O, or B). Solution: boundary condition (-rz), or $\langle M \rangle = 0$