Incorporating χ^2 in `sharp_combine`

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For any *re-sampled* pixel location j, `sharp_combine` calculates the mean intensity as

$$\langle I \rangle_j = \left[\sum_{k=1}^{M} \sum_{i=1}^{N_k} \; f_{i-j}^k \, w_i^k \, d_i^k \right] \; \left[\sum_{k=1}^{M} \sum_{i=1}^{N_k} \; f_{i-j}^k \, w_i^k \right]^{-1} , \tag{1}$$

where d_i^k is a measured intensity at a specific array pixel i in a specific data file k, $w_i^k = (\sigma_i^k)^{-2}$ is standard gaussian uncertainty weighting on the measurement of d_i^k, and $f_{i-j}^k = \exp\left(-\frac{r_{i-j}^2}{2\sigma_g^2} \right)$ is the gaussian smoothing kernel. The sums $i = 1 \rightarrow N_k$ is taken to be over all array locations in file k and then over all M files. Note that $f_{i-j}^k = 0$ for $r_{i-j} > m$, where m is the mask radius.

Using standard error propagation we find the variance on $\langle I \rangle_j$ is

$$\sigma_j^2 = \left[\sum_{k=1}^{M} \sum_{i=1}^{N_k} \; \left(f_{i-j}^k \right)^2 \, w_i^k \right] \; \left[\sum_{k=1}^{M} \sum_{i=1}^{N_k} \; f_{i-j}^k \, w_i^k \right]^{-2} . \tag{2}$$

To calculate the reduced-χ^2 we start with the defintion in Bevington's equation (11.4), using the first equality in equation (11.1) for the defintion of s^2 and equation (11.5) for $\langle \sigma_i^2 \rangle$. With these substitutions, Bevington's equation (11.4) becomes:

$$\chi_r^2 = \frac{1}{N-m} \sum \frac{[y_i - y(x_i)]^2}{\sigma_i^2} \tag{3}$$

where $N - m$ is the number of degrees of freedom.

If we consider that the "weighting" done in our equation (1) is the product $\left(f_{i-j}^k \, w_i^k\right)$ then we replace Bevington's $1/\sigma_i^2$ weighting with this weighting function to yield

$$\chi_j^2 = \left[\left(\sum_{k=1}^{M} N_k\right) - 1\right]^{-1} \left[\sum_{k=1}^{M}\sum_{i=1}^{N_k} f_{i-j}^k w_i^k \left(d_i^k - \langle I \rangle_j\right)^2\right] \tag{4}$$

Note we have also made the substitutions $y_i \to d_i^k$, $y(x_i) \to \langle I \rangle_j$, and the prefactor in square brackets is the number of degrees of freedom.

I am starting to think that a better version of equation (4) is

$$\begin{aligned} \chi_j^2 &= \left[\left(\sum_{k=1}^{M} N_k\right)\right]\left[\left(\sum_{k=1}^{M} N_k\right) - 1\right]^{-1} \times \\ & \left[\left(\sum_{k=1}^{M}\sum_{i=1}^{N_k} f_{i-j}^k\right)\right]^{-1} \left[\sum_{k=1}^{M}\sum_{i=1}^{N_k} f_{i-j}^k w_i^k \left(d_i^k - \langle I \rangle_j\right)^2\right]. \end{aligned} \tag{5}$$

Consider that the prefactor in equation (4) comes from, approximately,

$$\left[\left(\sum_{k=1}^{M}\sum_{i=1}^{N_k} w_i^k\right)\right]^{-1}. \tag{6}$$

where all $w_i^k = 1$. Since we know that the gaussian weighting factors f_{i-j}^k are not randomly distributed, we don't expect them to be uniform. So the correct normalization factor has to take that into account, as I tried to do in the first factor on the bottom line of equation (5). The factor on the top line of equation (5) takes into account that the sum over the f_{i-j}^k does not take into account the number of degrees of freedom.