The Astrophysical Journal, 616:L111–L114, 2004 December 1

EVALUATING THE MAGNETIC FIELD STRENGTH IN MOLECULAR CLOUDS

Martin Houde
Department of Physics and Astronomy, University of Western Ontario, London, ON N6A 3K7, Canada;
and Caltech Submillimeter Observatory, 111 Nowelo Street, Hilo, HI 96720; houde@astro.uwo.ca
Received 2004 September 7; accepted 2004 October 12; published 2004 October 21

ABSTRACT

We discuss an extension to the Chandrasekhar-Fermi method for the evaluation of the mean magnetic field strength in molecular clouds to cases where the spatial orientation of the field is known. We apply the results to M17, using previously published data.

Subject headings: ISM: clouds — ISM: individual (M17) — ISM: magnetic fields — polarization

1. INTRODUCTION

There exist few techniques that allow for the measurement of quantities that characterize the magnetic field in molecular clouds. At millimeter and submillimeter wavelengths, the orientation of the magnetic field is most commonly traced using polarimetry measurements from dust continuum emission (Hildebrand 1988). The strength of the magnetic field (in general, its line-of-sight component) can only be directly measured via the Zeeman effect (e.g., Crutcher et al. 1999; Brogan & Troland 2001), usually at longer wavelengths. In order to gather as much information as possible about the magnetic field, the so-called Chandrasekhar-Fermi (CF) method (Chandrasekhar & Fermi 1953) is often used to infer the strength of the plane-of-the-sky component of the field. Because this is achieved with the same polarimetry data that give the orientation of the sky-projected magnetic field, the CF method can act as a bridge between the polarimetry and Zeeman observations to provide an estimate for the magnitude of the mean field strength in a given cloud.

In this Letter, we discuss how a simple extension of the CF method can be used alone, i.e., without the need of Zeeman data, to infer the magnitude of the magnetic field (and not only the strength of its plane-of-the-sky component). Furthermore, it will also be shown that, contrary to the original CF method, which only really works well when the magnetic field is located close enough to the plane of the sky, our generalization is valid regardless of the field's orientation in space. However, this can only be accomplished if and when the spatial orientation of the magnetic field is known. That is to say, not only the orientation of its projection on the plane of the sky is needed (from polarimetry), but also its inclination to the line of sight. This last piece of information can be obtained through the technique of Houde et al. (2002), which relies on the availability of spectroscopic measurements from suitable neutral and ionic molecular species, as well as polarimetry.

Finally, we apply our extension to the CF method to already published data for the M17 molecular cloud (Houde et al. 2002) and infer a value for the magnitude of the mean magnetic field for this object.

2. THE CF EQUATION

It was originally asserted by Chandrasekhar & Fermi (1953) that the amount of dispersion of the polarization angles measured from starlight (or dust continuum radiation) can reveal information about the magnitude of the magnetic field. With the assumption that the magnetic field is frozen to the ambient fluid, any (turbulent) motion within the gas in a direction perpendicular to the orientation of the magnetic field will be transmitted to, and distort, the field lines. Chandrasekhar & Fermi (1953) further assumed that such disturbances would propagate as waves along the magnetic field lines at the Alfvén speed, which they used as the starting point for their analysis. It follows that since dust grains are thought to be tied to the magnetic field lines (Mouschovias & Ciolek 1999), the amount of distortion in the field lines can be inferred from polarimetry. Similarly, the turbulent motion of the gas can be measured through the spectral line profiles of molecular species, for example. These two observed quantities are needed to evaluate the strength of the magnetic field through the CF method.

Following, therefore, the original derivation of Chandrasekhar & Fermi (1953), we can write an equation for the mean value of the magnetic field B as

$$B = \sqrt{4\pi\rho}\,\frac{\sigma(v_\perp)}{\sigma(\phi)}, \tag{1}$$

where ρ and $\sigma(v_\perp)$ are, respectively, the mass density and the two-dimensional velocity dispersion (perpendicular to the field lines) of the matter coupled to the magnetic field, and $\sigma(\phi)$ is the dispersion in angular deviations of the field lines. Equation (1) is valid in the small-angle limit.

In their estimation of the magnetic field strength in the spiral arms, Chandrasekhar & Fermi (1953) identified $\sigma(\phi)$ with the dispersion in the orientation of the polarization vectors measured for distant background stars. Using the coordinate system of Figure 1 to define the spatial orientation of the magnetic field, with α the inclination angle of the field to the line of sight and β the angle made by its projection on the plane of the sky, we find, for the case originally considered by Chandrasekhar & Fermi (1953), that

$$\sigma(\phi) = \sigma(\beta). \tag{2}$$

However, observations of this type probe only one direction in the lateral displacement of the magnetic field lines. We must, therefore, make the following substitution for the velocity dispersion:

$$\sigma(v_\perp) \to \frac{1}{\sqrt{2}}\,\sigma(v_\perp) = \frac{1}{\sqrt{3}}\,\sigma(v), \tag{3}$$

where $\sigma(v)$ is the total three-dimensional velocity dispersion of the gas (for cases of isotropic turbulence). Inserting equations (2)

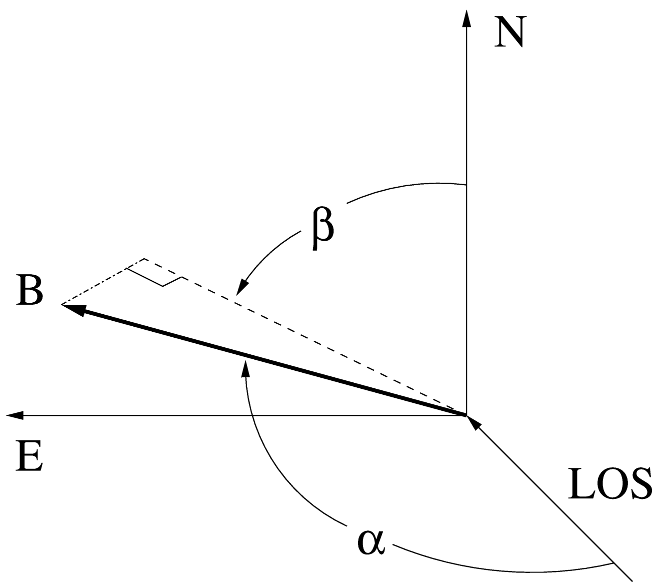

FIG. 1.—The spatial orientation of the magnetic field is defined with the two angles α and β. The N, E, and LOS axes stand for north, east, and line of sight, respectively (from Houde et al. 2002).

and (3) in equation (1), we obtain the original equation derived by Chandrasekhar & Fermi (1953):

$$B_{\rm pos} = \sqrt{\frac{4}{3}\pi\rho}\,\frac{\sigma(v)}{\sigma(\beta)}, \tag{4}$$

where $B_{\rm pos}$ is the plane-of-the-sky component of the magnetic field (more on this below).

Equation (4) is often used to measure the mean strength of the plane-of-the-sky component of the magnetic field in molecular clouds (e.g., Lai et al. 2003a). It has also been tested with magnetohydrodynamic (MHD) simulations to verify its domain of applicability (Ostriker et al. 2001; Padoan et al. 2001; Heitsch et al. 2001; Kudoh & Basu 2003). Although the CF method has been found to work well for a strong enough magnetic field, it also suffers from some shortcomings. Among these is the fact that equation (4) only really applies well when the magnetic field is located close enough to the plane of the sky. In fact, the method will fail when the field is aligned parallel to the line of sight ($\alpha = 0$ in Fig. 1).

2.1. *An Extension to the CF Method*

It would be desirable to extend the CF method to cases where the magnetic field is arbitrarily oriented in space. This, however, requires that observations can be made to measure not only β (the angle made by the projection of the magnetic field on the plane of the sky) but also α (the inclination angle of the field to the line of sight). Some methods have already been proposed to do such measurements. Myers & Goodman (1991; see also Bourke & Goodman 2004) modeled the magnetic field in molecular clouds with uniform and nonuniform components, and through a statistical analysis they were able to evaluate the spatial orientation (i.e., they inferred α and β) for the mean three-dimensional uniform field. More recently, Houde et al. (2002) have proposed a technique that combines polarimetry and ion–to–neutral line width ratio measurements (Houde et al. 2000a, 2000b) to map the spatial orientation of the magnetic field across molecular clouds. This method has been used so far for three different objects: M17 (Houde et al. 2002), DR 21(OH) (Lai et al. 2003b), and Orion A (Houde et al. 2004).

Once α and β are mapped across a given molecular cloud, the angular dispersions $\sigma(\alpha)$ and $\sigma(\beta)$ can be calculated from the measured data. It is easy to show that, in the small-angle limit, the total angular dispersion of the magnetic field lines $\sigma(\phi)$ is given by

$$\sigma^2(\phi) = \sigma^2(\alpha) + \sin^2(\alpha)\sigma^2(\beta). \tag{5}$$

Equation (5) takes into account not only the inclination of the magnetic field but also angular deviations along two independent directions perpendicular to the field orientation. Because of this last point, the velocity dispersion will be $\sqrt{2}$ times larger than what is used in the original CF method (eq. [4]). That is to say, we will now use either the two-dimensional velocity dispersion $\sigma(v_\perp)$, defined after equation (1), or its equivalent expressed as a function of $\sigma(v)$ if the turbulence is isotropic:

$$\sigma(v_\perp) = \sqrt{\frac{2}{3}}\,\sigma(v). \tag{6}$$

Using equations (5) and (6), we can now write a generalized CF equation from equation (1),

$$B = C\left[\frac{4\pi\rho\sigma^2(v_\perp)}{\sigma^2(\alpha) + \sin^2(\alpha)\sigma^2(\beta)}\right]^{1/2}, \tag{7}$$

or if the turbulence is isotropic,

$$B = C\left\{\frac{8\pi\rho\sigma^2(v)}{3[\sigma^2(\alpha) + \sin^2(\alpha)\sigma^2(\beta)]}\right\}^{1/2}. \tag{8}$$

In both equations (7) and (8) we have added a correction factor C (first introduced by Ostriker et al. 2001) to take into account some shortcomings of the CF method to be discussed later. It is now easy to see how equation (8) can be reduced to one for the plane-of-the-sky component of the magnetic field $B_{\rm pos}$ (i.e., eq. [4]) when only polarization measurements are available. In this case, for a sufficiently large set of data we expect (as long as $\alpha \neq 0$)

$$\sigma^2(\alpha) = \sin^2(\alpha)\sigma^2(\beta)$$

and

$$\sigma^2(\phi) = 2\sin^2(\alpha)\sigma^2(\beta).$$

We can write

$$B_{\rm pos} = B\sin(\alpha) = \sqrt{\frac{4}{3}\pi\rho}\,\frac{\sigma(v)}{\sigma(\beta)},$$

which is the same as equation (4).

We can therefore emphasize two important advantages of the modified CF equation (7) (or eq. [8]) over the original:

1. The new equation is valid no matter what the orientation

of the magnetic field is. Most notably, the method does not fail when the field is directed along the line of sight.

2. Finally, the value for the magnetic field calculated with equation (7) is not that of its plane-of-the-sky component but is the *full magnitude of the mean magnetic field vector*.

2.2. *Shortcomings of the Method*

As mentioned earlier, MHD simulations have already been used in the past (Ostriker et al. 2001; Padoan et al. 2001; Heitsch et al. 2001; Kudoh & Basu 2003) to test the validity of the original CF method (eq. [4]). The main conclusion of these studies was that the introduction of a correction factor (C in eqs. [7] and [8]) is needed when evaluating B_{pos}. A correction of $C \sim 0.5$ was deemed appropriate in most cases when the field is not too weak. A few reasons are usually identified for this. For example:

1. Because of the finite resolution with which observations are done, there will be an averaging of the angular structure of the field (i.e., a smoothing of the field). This will bring a decrease of the angular dispersion $\sigma(\phi)$, and an overestimation of the field strength (Ostriker et al. 2001).
2. Similarly, line-of-sight averaging (independent of the angular resolution of the observations) of the magnetic field will decrease $\sigma(\phi)$ (Myers & Goodman 1991).
3. Inhomogeneity and complex density structures (e.g., clumpiness) also tend to reduce the value of C (Zweibel 1990; Ostriker et al. 2001).

We also add to the previous points one more aspect that should be kept in mind when applying the CF method. In the case of highly turbulent and massive molecular clouds (like in the example considered in the next section), it has been observed that there can exist significant velocity drifts between coexistent neutral and ionic molecular species. This can be ascertained through the comparison of the observed line profiles for the two types of species, with the ions consistently exhibiting narrower spectral line widths (Houde et al. 2000a, 2000b, 2002, 2004; Lai et al. 2003b). This implies that the coupling between ions and neutrals is not perfect (Houde et al. 2002). Within the context of the CF method, this brings about uncertainties in two of the quantities used when evaluating the magnetic field strength. Indeed, because of this imperfect coupling between ions and neutrals, the mass density ρ used in the CF equation cannot be that of (larger) neutral density. It must be somewhat smaller. Furthermore, because of the aforementioned velocity drift, the velocity dispersion perpendicular to the field lines $\sigma(v_\perp)$ [or $\sigma(v)$] cannot be that measured for a neutral molecular species. It must also be smaller. The combination of these factors will also tend to reduce the value of C (in eq. [7] or eq. [8]), at least when the CF method is applied to highly turbulent and massive molecular clouds. We leave the quantification of these effects as open questions that could, perhaps, be investigated through simulations.

3. APPLICATION OF THE EXTENDED CF METHOD TO M17

Using their aforementioned technique, Houde et al. (2002) measured the spatial orientation of the magnetic field at 57 different positions across the M17 molecular clouds. This was accomplished using extensive 350 μm dust continuum polarimetry and spectroscopy (HCO^+/HCN) maps obtained at the Caltech Submillimeter Observatory. We now use their results to calculate the mean magnetic field strength for M17, using equation (7).[1]

From the analysis of Houde et al. (2002) we find the following averages for M17:

$$\begin{aligned} \alpha &\simeq 47\overset{\circ}{.}1, \\ \sigma(\alpha) &\simeq 10\overset{\circ}{.}8, \\ \beta &\simeq 76\overset{\circ}{.}1, \\ \sigma(\beta) &\simeq 16\overset{\circ}{.}7 \\ \sigma(\phi) &\simeq 16\overset{\circ}{.}3, \\ \sigma(v_\perp) &\simeq 2.0\ \mathrm{km\ s^{-1}}. \end{aligned}$$

The transverse velocity dispersion was evaluated from the HCN spectra, taking into account the (anisotropic) turbulent flow model used by Houde et al. (2002; see their Fig. 2 and eq. [11]) and the fact that the inclination angle is known.[2] Upon using equation (7) with $C = 0.5$, an approximate value of $10^6\ \mathrm{cm^{-3}}$ for the mean density, and a mean molecular mass of 2.3, we find

$$B \approx 2.5\ \mathrm{mG}.$$

This value for the magnitude of the magnetic field could be further reduced if the correction factor C were found to be smaller than the stated value (because of the effects discussed in the last paragraph of § 2.2) or, again, if the average density across the maps were less than what was assumed here. However, this field strength may not be too excessive in light of the fact that Brogan & Troland (2001) obtained a peak value of $-750\ \mu$G for the line-of-sight component of the magnetic field in M17, using H I Zeeman measurements. For once the inclination angle quoted above is taken into account, we calculate from their data a field magnitude in excess of 1 mG. Our molecular species (i.e., HCN and HCO^+, in the $J = 4 \rightarrow 3$ transition) probe denser media that could harbor stronger fields.

It is also interesting to note that

$$\sigma(\alpha) \sim \sin(\alpha)\sigma(\beta) = 12\overset{\circ}{.}2,$$

as would be expected for a large enough data set.

Finally, we would like to state that the extension to the CF method presented in this Letter should be readily testable through MHD simulations, as was done in the past for the original CF technique (Ostriker et al. 2001; Padoan et al. 2001; Heitsch et al. 2001; Kudoh & Basu 2003).

The author thanks T. G. Phillips, R. Peng, and S. Basu for helpful discussions. The Caltech Submillimeter Observatory is funded by the NSF through contract AST 99-80846.

[1] The values for α and β used here are slightly different from those presented in Houde et al. (2002). We use a maximum polarization level of 10%, instead of 7% as was used in their original analysis. See Houde et al. (2004) for more details.

[2] Within the context of the anisotropic turbulent model of Houde et al. (2002), a value for $\sigma(v_\perp)$ at each position can be obtained from the corresponding observed spectral line width $\sigma_{\mathrm{obs}}(v)$. It can be shown that $\sigma^2(v_\perp) = \sigma^2_{\mathrm{obs}}(v) f / [e\cos^2(\alpha) + f/2 \sin^2(\alpha)]$, where e and f are given in their eq. (11) with $\Delta\theta = 44\overset{\circ}{.}4$.

REFERENCES

Bourke, T. L., & Goodman, A. A. 2004, in IAU Symp. 221, Star Formation at High Angular Resolution, ed. M. Burton, R. Jayawardhana, & T. Bourke (San Francisco: ASP), in press (astro-ph/0401281)

Brogan, C. L., & Troland, T. H. 2001, ApJ, 560, 821

Chandrasekhar, S., & Fermi, E. 1953, ApJ, 118, 113

Crutcher, R. M., Troland, T. H., Lazareff, B., Paubert, G., & Kazès, I. 1999, ApJ, 514, L121

Heitsch, F., Zweibel, E. G., Mac Low, M.-M., Li, P., & Norman, M. L. 2001, ApJ, 561, 800

Hildebrand, R. H. 1988, QJRAS, 29, 327

Houde, M., et al. 2002, ApJ, 569, 803

Houde, M., Bastien, P., Peng, R., Phillips, T. G., & Yoshida, H. 2000a, ApJ, 536, 857

Houde, M., Dowell, C. D., Hildebrand, R. H., Dotson, J. L., Vaillancourt, J. E., Phillips, T. G., Peng, R., & Bastien, P. 2004, ApJ, 604, 717

Houde, M., Peng, R., Phillips, T. G., Bastien, P., & Yoshida, H. 2000b, ApJ, 537, 245

Kudoh, T., & Basu, S. 2003, ApJ, 595, 842

Lai, S.-P., Girart, J. M., & Crutcher, R. M. 2003a, ApJ, 598, 392

Lai, S.-P., Velusamy, T., & Langer, W. D. 2003b, ApJ, 596, L239

Mouschovias, T. C., & Ciolek, G. E. 1999, in The Origins of Stars and Planetary Systems, ed. C. J. Lada & N. D. Kylafis (Dordrecht: Kluwer), 305

Myers, P. C., & Goodman, A. A. 1991, ApJ, 373, 509

Ostriker, E. C., Stone, J. M., & Gammie, C. F. 2001, ApJ, 546, 980

Padoan, P., Goodman, A. A., Draine, B. T., Juvela, M., Nordlund, Å., & Rögnvaldsson, Ö. E. 2001, ApJ, 559, 1005

Zweibel, E. G. 1990, ApJ, 362, 545